CBSE SAMPLE PAPER-01

CBSE Class – XI

MATHEMATICS

Time allowed: 3 hours,Maximum Marks: 100

General Instructions:

a) All questions are compulsory.

b) The question paper consists of 26 questions divided into three sections A, B and C. Section A comprises of 6 questions of one mark each, Section B comprises of 13 questions of four marks each and Section C comprises of 7 questions of six marks each.

c) All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

d) Use of calculators is not permitted.

Section A

1. **Find the number of subsets of a set A containing 10 elements.**

Sol: Number of subsets

$^{10}C_0 + {}^{10}C_1 + {}^{10}C_2 + {}^{10}C_3 + {}^{10}C_4 + {}^{10}C_5 + {}^{10}C_6 + {}^{10}C_7 + {}^{10}C_8 + {}^{10}C_9 + {}^{10}C_{10} = 2^{10}$

2. **How many ways can you choose one or more students from 3 students?**

Sol: $^3C_1 + {}^3C_2 + {}^3C_3 + = 2^3 - 1 = 7$

3. In How many ways can one choose 3 cards from a pack of 52 cards in succession (1) with replacement (2) without replacement?

Sol: (1)Each card can be drawn in 52 ways and so the total number of ways

$52 \times 52 \times 52 = 52^3$

(2)If there is no replacement the first card can be drawn in 52 ways, the second by 51 ways and the third by 50 ways. Hence the total number of ways is

$52 \times 51 \times 50 = 132600$

4. State the condition under which the product of two complex numbers is purely imaginary.

Sol: 1.None of the factors are zero

2.Factors must be of the form $(a+ib); k(b+ia)$ where k is a real number.

5. In a circle of radius 1 unit what is the length of the arc that submits an angle of 2 radians at the centre.

Sol: Length of arc = $r\theta$

Hence length of arc==2units

6. **Is $\cos\theta$ positive or negative if $\theta = 500\, radians$.**

Sol: 1 Full rotation is $2\pi radians$

500 radians = $\frac{500}{2\pi}$ rotations

$\frac{500}{2\pi} = 79.57 rotations$

79 full rotations and 0.57 of a rotation

$0.5 < 0.57 < 0.75$

The incomplete rotation is between $\frac{1}{2} and \frac{3}{4}$ of a rotation . Hence 500 radians is in third quadrant. So $\cos\theta$ is negative

Section B

7. Prove by mathematical induction that $n(n+1)(2n+1)$ is divisible by 6 if n is a natural number.

Sol: Let $n = 1$

Then $n(n+1)(2n+1)$ = 6 and divisible by 6

Let it be divisible by 6 for $n = m$

Then $m(m+1)(2m+1) = 6k$ Where k is an integer

For $n = m+1$ the expression is

(m + 1)(m + 2)(2m + 2 + 1) = (m + 2)(m + 1)(2m + 1) + 2(m + 1)(m + 2)

=m(m + 1)(2m + 1) + 2(m + 1)(2m + 1) + 2(m + 1)(m + 2)

=m(m + 1)(2m + 1) + 2(m + 1)(3m + 3)

=m(m + 1)(2m + 1) + 6(m + 1)^2

=6k + 6(m + 1)^2, This is divisible by 6.

8. Solve $\cos 2x - 5\sin x - 3 = 0$.

Sol: $1 - 2\sin^2 x - 5\sin x - 3 = 0$

$2sin^2 x + 5\sin x + 2 = 0$

Let $sinx = t$

Then, $2t^2 + 5t + 2 =$
0 Solving this quadratic

$2t(t+2) + (t+2) = 0$

$(2t+1)(t+2) = 0$

$t = -2, t = -\frac{1}{2}$

$\sin x = \frac{-1}{2}$

First value of t is rejected as $\sin x$ should lie between $(-1 \quad and \quad 1)$

General solution is $x = (-1)^{n+1}\frac{\pi}{6} + n\pi$

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9. **For what values of $m^2x^2 + 2(m + 1)x + 4 = 0$ will have exactly one zero.**

Sol: When $m = 0$

The given equation reduces to a first degree and it will have only one solution

Also when the discriminant is zero it will have only one solution

Discriminant is

$4(m+1)^2 - 4m^2.4 = 0$

$4(m^2 + 1 + 2m) - 16m^2 = 0$

On simplifying and solving,

$(m-1)(3m+1) = 0$

$m = 1, m = -\frac{1}{3}$

Hence the three values of m for which the equation will have only one solution is

$m = 0, m = 1, m = -\frac{1}{3}$

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10. Three numbers are in AP. Another 3 numbers are in GP. The sum of first term of the AP and the first term of the GP is 85, the sum of second term of AP and the second term of the GP is 76 and that of the 3rd term of AP and 3rd term of GP is 84. The sum of the AP is 126. Find each term of AP and GP.

Sol:

A.P	a-d, a, a+d
G.P	b/g, b, bg

$$a-d+\frac{b}{g}=85\ldots(1)$$
$$a+d+bg=84\ldots(2)$$
$$2a+\frac{b}{g}+bg=169$$
$$34g^2-85g+34=0$$
$$g=\frac{85\pm\sqrt{85^2-4\times34\times34}}{2\times34}$$
$$g=2\quad or\quad \frac{1}{2}$$

When g = 2

$42 - d + \frac{34}{2} = 85$

d = - 26

a = 42, d = - 26, g = 2, b = 34

AP

$68,\ \ 42,\ \ 16$

GP

$17,\ \ 34,\ \ 68$

$m=1, m=-\frac{1}{3}$

11. If $f(x)=4^x$ find $f(x+1)-f(x)$ in terms of $f(x)$.

Sol: $f(x+1)=4^{x+1}$

$f(x)=4^x$

$f(x+1)-f(x)$

$=4^{x+1}-4^x$

$=4^x.4-4^x$

$=4^x(3)$

$=3f(x)$

12. If $f(x)=\log\frac{(1+x)}{(1-x)}$ Prove that

$$f\left(\frac{3x+x^3}{1+3x^2}\right) = 3f(x) \text{ when } \quad -1 < x < 1$$

$$f\left(\frac{3x+x^3}{1+3x^2}\right) = 3f(x)$$

$when \quad -1 < x < 1$

Sol:

$$\log \frac{1+\frac{3x+x^3}{1+3x^2}}{1-\frac{3x+x^3}{1+3x^2}}$$

$$= \log \frac{1+3x^2+3x+x^3}{1+3x^2-3x-x^3}$$

$$= \log \frac{(1+x)^3}{(1-x)^3}$$

$$= 3\log \frac{(1+x)}{(1-x)}$$

$$= 3f(x)$$

13. Find the value of $\sin 75 and \cos 75$ **.**

Sol:

$\sin(45+30) = \sin 45 \cos 30 + \cos 45 \sin 30$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}+1}{2\sqrt{2}}$

$= \frac{\sqrt{6}+\sqrt{2}}{4}$

$\cos(45+30) = \cos 45 \cos 30 - \sin 45 \sin 30$

$= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} - \frac{1}{\sqrt{2}} \times \frac{1}{2}$

$= \frac{\sqrt{3}-1}{2\sqrt{2}}$

$= \frac{\sqrt{6}-\sqrt{2}}{4}$

14. Prove that $\frac{sin3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = 2$.

Sol:

$$\frac{sin3\theta}{\sin\theta} - \frac{\cos 3\theta}{\cos\theta} = \frac{\sin 3\theta\cos\theta - \cos 3\theta\sin\theta}{\sin\theta\cos\theta}$$
$$= \frac{\sin(3\theta-\theta)}{\sin\theta\cos\theta}$$
$$= \frac{2\sin 2\theta}{2\sin\theta\cos\theta}$$
$$= \frac{2\sin 2\theta}{\sin 2\theta} = 2$$

15. If the line $y = mx + 1$ is a tangent to the ellipse $x^2 + 4y^2 = 1$ then find the value of m^2.

Sol:

$$x^2 + 4(mx+1)^2 = 1$$
$$x^2 + 4(m^2x^2 + 2mx + 1) = 1$$
$$x^2 + 4m^2x^2 + 8mx + 4 = 1$$
$$x^2(1 + 4m^2) + 8mx + 3 = 0$$

The line being a tangent ,it touches the ellipse at two coincident points, and so Discriminant must be zero,

$(8m)^2 - 4(3)(1 + 4m^2) = 0$

$64m^2 - 12 - 48m^2 = 0$

$16m^2 = 12$

$m^2 = \frac{12}{16}$

$m^2 = \frac{3}{4}$

16. **Reduce the equation** $3x - 4y + 20 = 0$ **in to normal form.**

Sol: Divide the equation by $-\sqrt{3^2 + -4^2} = -5$

Hence, $-\frac{3}{5}x + \frac{4}{5}y - 4 = 0$

Where, cos $\alpha = \frac{-3}{5}$ and sin $\alpha = \frac{4}{5}$ and p=4

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17. Solve the inequality $\frac{x+3}{x-7} \leq 0$.

Sol: Multiply both numerator and denominator with $x - 7$.Then denominator becomes a perfect square and it is always positive

Now $(x + 3)(x - 7) \leq 0$

Critical points are $(-3, 7)$

Hence, $-3 \leq x < 7$

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18. Find $\lim_{x\to\infty} \frac{x^2 - ax + 4}{3x^2 - bx + 7}$

Sol:

$$\lim_{x\to\infty} \frac{x^2 - ax + 4}{3x^2 - bx + 7}$$

$$= \lim_{x\to\infty} \frac{x^2(1 - \frac{a}{x} + \frac{4}{x^2})}{x^2(3 - \frac{b}{x} + \frac{7}{x^2})}$$

= $\frac{1}{3}$

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19. Find $\lim_{x\to 0} \frac{\tan x}{\sin 3x}$

Sol:

$$\lim_{x\to 0} \frac{\tan x}{\sin 3x}$$

$$= \lim_{x\to 0} \frac{\sin x}{x} \times \frac{1}{cosx} \times \frac{1}{\frac{3sin3x}{3x}}$$

$= 1 \times 1 \times \frac{1}{3} = \frac{1}{3}$

Section C

20. **Evaluate $x^3 + x^2 - 4x + 13$ when $x = 1 + i$ and when $x = 1 - i$**

Sol: Form a quadratic equation whose roots are $1+\ i\ \ and\ \ \ 1\ -i$

The equation is $x^2 - 2x + 2 = 0$

The given expression

$x^3 + x^2 - 4x + 13 = x(x^2 - 2x + 2) + 3(x^2 - 2x + 2) + 7$

$x^3 + x^2 - 4x + 13 = x(0) + (0) + 7$

$x^3 + x^2 - 4x + 13 = 7$

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21. Prove that the roots of the equation $(x - \alpha)(x - \beta) = k^2$ is always real.

Sol: $x^2 - (\alpha + \beta)x + \alpha\beta - k^2 = 0$

Discriminant of the above quadratic is $\{(\alpha + \beta)\}^2 - 4(\alpha\beta - k^2)$

$= (\alpha - \beta)^2 + k^2$ is always positive and hence the roots are real.

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22. If the roots of the equation $lx^2 + nx + n = 0$ are in the ratio $p : q$ then prove that

$$\frac{\sqrt{p}}{\sqrt{q}} + \frac{\sqrt{q}}{\sqrt{p}} + \frac{\sqrt{n}}{\sqrt{l}} = 0 \cdot$$

Sol: Let the roots be $p\alpha\quad and\quad q\beta$

Then $p\alpha + q\alpha = -\frac{n}{l}\ldots(1)$

$pq\alpha^2 = \frac{n}{l}$

$\alpha = \frac{\sqrt{n}}{\sqrt{l}} \times \frac{1}{\sqrt{pq}}\ldots(2)$

Hence substituting equation 2 in equation 1

$(p + q)\frac{\sqrt{n}}{\sqrt{l}} \times \frac{1}{\sqrt{pq}} + \frac{n}{l} = 0$

On simplifying ,

$\frac{\sqrt{p}}{\sqrt{q}} + \frac{\sqrt{q}}{\sqrt{p}} + \frac{\sqrt{n}}{\sqrt{l}} = 0$

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23. Find $\lim_{x\to\pi}(\pi - x)\tan\frac{x}{2}$**.**

Sol:

$$= \lim_{\frac{x-\pi}{2} \to 0} 2 \frac{\cos \frac{\pi - x}{2}}{\frac{\sin \frac{\pi - x}{2}}{\frac{\pi - x}{2}}}$$

$$= \lim_{\frac{x-\pi}{2} \to 0} 2 \frac{\cos \frac{x - \pi}{2}}{\frac{\sin \frac{x - \pi}{2}}{\frac{x - \pi}{2}}} = 2$$

since the limit of $\frac{\sin \frac{x-\pi}{2}}{\frac{x-\pi}{2}} = 1$

24. **If a, b, c are 3 consecutive integers prove that (a - i)(a + i)(c + i)(c - i) = b^4 + 1.**

Sol: Let $a = x - 1$

$b = x$

$c = x + 1$

Then $(x - 1 - i)((x - 1 + i)(x + 1 + i)(x + 1 - i)$

$= \{(x-1)^2 - i^2\}\{(x+1)^2 - i^2\}$

$= \{(x-1)^2 + 1\}\{(x+1)^2 + 1\}$

$= \{(x-1)(x+1)\}^2 + (x-1)^2 + (x+1)^2 + 1$

$= (x^2 - 1)^2 + (x-1)^2 + (x+1)^2 + 1$

$= x^4 + 1$

$= b^4 + 1$

25. Prove that $\frac{(1+i)^n}{(1-i)^{n-2}} = 2\, i^{n-1}$.

Sol: Multiply both Numerator and denominator with $(1-i)^2$. Then

$\frac{(1+i)^n}{(1-i)^{n-2}} = \frac{(1+i)^n (1-i)^2}{(1-i)^n}$

multiplying both Numerator & denominator with $(1+i)^n$

$= \frac{(1+i)^n (-2i)(1+i)^n}{(1-i)^n (1+i)^n}$

Simplifying

$= \frac{\{(1+i)^2\}^n(-2i)}{(1-i^2)^n}$

On expanding and simplifying

$= \frac{2^n i^n(-2)i}{2^n}$

$= -2i^{n+1}$

$= \frac{2(i)^{n+1}}{i^2}$

$= 2i^{n-1}$

26. Determine the coordinates of a point which is equidistant from the point $(1, 2) and (3, 4)$ and the shortest distance from the line joining the points $(1, 2) and (3, 4)$ to the required point is $\sqrt{2}$.

Sol: Let the point be A(1, 2) and B(3, 4)

The mid-point of the line joining A and B is $C(2, 3)$

Slope of line AB = $\frac{4-2}{3-1} = 1$

Let the required point be $D(\alpha, \beta)$

Then D must be a point on the line perpendicular to the line AB and passing through point C

$\therefore$ Slope of CD = - 1

Equation of CD

y - 3 = - 1(x - 2)

x + y = 5

Equation of AB

y - 2 = 1(x - 1)

x - y + 1 = 0

The point $D(\alpha, \beta)$ must satisfy the equation

x + y = 5

$\therefore \alpha + \beta = 5 \ldots (1)$

The perpendicular distance from (α, β) to AB is

$\frac{\alpha-\beta+1}{\sqrt{2}} = \sqrt{2}$

$\alpha - \beta = 1 \ldots (2)$

Solving equations 1 and 2

$\alpha = 3, \beta = 2$